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Alternative Inflationary Scenario Due to Compact Extra Dimensions

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Abstract The main goal of this paper is to give an alternative interpretation of space-like and time-like extra dimensions as a primary factor for inflation in the early universe. We introduce the 5-dimensional perfect fluid and compare the energy-momentum tensor for the bulk scalar field with space-like and time-like extra dimensions. It is shown, that additional dimensions can imply to negative pressure in the slow roll regime in the early higher-dimensional world.

Keywords Inflation · 5D Metric · Negative pressure

1 Introduction

The 5-dimensional warped geometry theory is a braneworld theory developed by Physicists, Lisa Randall and Ramam Sundrum, while trying to solve the hierarchy problem of the Standard Model [1–5]. They consider one extra "non-factorizable" dimension. The metric is assumed to be

$$ds^2 = g_{ab}dx^a dx^b = \eta_{\mu\nu}e^{-2kry}dx^\mu dx^\nu + r^2 dy^2, \tag{1}$$

which $a, b = (0, 1, 2, 3, 4)$ and Greek indices $\mu, \nu = (0, 1, 2, 3)$, where refer to the four observable dimensions, and also y signifies the coordinate on the additional dimension of length r , and the factor e^{-2kry} is called warp factor. The metric tensor in this model given by

$$g_{ab} = diag(e^{-2A(y)}, -e^{-2A(y)}, -e^{-2A(y)}, -e^{-2A(y)}, r^2), \tag{2}$$

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which $e^{-2A(y)}$ is called generalized warp factor [3]. We have for space-like extra dimensions (SLED), $r^2 = -1$ and for time-like extra dimensions (TLED), $r^2 = +1$. Very often, the standard tenet in dealing with higher-dimensional theories is to consider space-like extra dimensions. But time-like extra dimensions have been disregarded due to the serious conflicts with causality and unitarity [6–8].

Hear it was assumed to have two branes. One at $y = 0$, called the Planckbrane (where gravity is a relatively strong force; also called Gravitybrane), and one at $y = r\pi$, called the Tevbrane (our home with the Standard Model particles; also called Weakbrane). In that case, Tevbrane tells us that all standard fields are assume to live on the second brane. By warping any Lagrangian mass parameter which is naturally $\approx M_{PL}$, will appear to us on the S.M. brane to be $\approx$TeV. Thus by considering warped extra dimensions one is able to solve the hierarchy problem. This means that transition from 4D world to 5D world, exponentially shrink size and grow mass and energy [1, 2, 9–11].

In this paper we have considered opposite process of above, *i.e.* transition from 5D world to the 4D of it. By consideration of symmetry, it can be expected exponentially expansions of size occur in this transition. Therefore, we can foretell the alternative inflationary scenario for 5D world before of the standard inflation in 4D world. Thus, the outline of paper is as follows: In Sect. 2, the energy-momentum tensor for 4D metric is recalled briefly. In Sect. 3, we introduce the 5D perfect fluid and compare of the energy-momentum tensor for the balk scalar field with space-like and time-like extra dimensions. Brief conclusions are given in final section.

2 The Energy-Momentum Tensor for 4D Metric

In this section, we consider a single scalar field, called *inflaton field* during the period of inflation. The Lagrangian of the inflaton field [12–16], ϕ, is

$$L = \frac{1}{2} g_{\mu\nu} \partial^\mu \phi \partial^\nu \phi - V(\phi), \tag{3}$$

where

$$g_{\mu\nu} = diag(a^2(\eta), -a^2(\eta), -a^2(\eta), -a^2(\eta)), \tag{4}$$

is the F.R.W. metric with the conformal time coordinate η, by the definition $dt = a(\eta)d\eta$. Also $a(\eta)$ is the scalar factor which depends only on time.

The action for the inflaton field is

$$S = \int d^4x \sqrt{-g} L = \int d^4x \sqrt{-g} \left[\frac{1}{2} g_{\mu\nu} \partial^\mu \phi \partial^\nu \phi - V(\phi) \right]. \tag{5}$$

The energy-momentum tensor for the inflaton field is given by [17],

$$T_{\mu\nu} = -g_{\mu\nu} L + \partial_\mu \phi \partial_\nu \phi. \tag{6}$$

For any perfect fluid, this tensor given by

$$T^\mu_\nu = diag(\rho, -P, -P, -P), \tag{7}$$

hear ρ and P are the density and pressure of the perfect fluid respectively. Remember that “perfect” can be taken to means “isotropic in its rest frame”. This in turn means that T^μ_ν

is diagonal—there is flux of any component of momentum in an orthogonal direction. Furthermore, the nonzero space-like components must all be equal, *i.e.* $T_1^1 = T_2^2 = T_3^3$ [17]. Because $T_{\mu\nu} = g_{\mu\beta} T_\nu^\beta$, one obtains

$$T_{\mu\nu} = diag(\rho a^2(\eta), Pa^2(\eta), Pa^2(\eta), Pa^2(\eta)). \tag{8}$$

Considering the inflaton field as a homogeneous perfect fluid, its energy-momentum tensor in (6) can be written in components as

$$T_{00} = \left[\frac{1}{2}\left(\frac{\partial\phi}{\partial\eta}\right)^2 + V(\phi)a^2(\eta)\right], \tag{9}$$

$$T_{0i} = 0, \tag{10}$$

$$T_{ij} = \left[\frac{1}{2}\left(\frac{\partial\phi}{\partial\eta}\right)^2 - V(\phi)a^2(\eta)\right]\delta_{ij}. \tag{11}$$

Comparing the results with (8), the energy density and the pressure of the inflaton field are

$$\rho = \left[\frac{1}{2a^2(\eta)}\left(\frac{\partial\phi}{\partial\eta}\right)^2 + V(\phi)\right], \tag{12}$$

$$P = \left[\frac{1}{2a^2(\eta)}\left(\frac{\partial\phi}{\partial\eta}\right)^2 - V(\phi)\right]. \tag{13}$$

It can be seen that when the potential energy of the inflaton field is larger than its Kinetic energy (slow-roll condition), the negative pressure appears. This state, *i.e.* $P = -\rho$ is very important in order to have inflation ($\ddot{a} > 0$).

3 The Energy-Momentum Tensor for 5D Metric

The Lagrangian of the scalar field, ϕ, in 5-dimensional world is

$$L = \frac{1}{2} g_{ab}\partial^a\phi\partial^b\phi - V(\phi). \tag{14}$$

Which the field $\phi = \phi(y)$, is a balk scalar field which respect to only the extra dimension coordinate y.

The general form of action is

$$S = \int d^5x\sqrt{-g}L = \int d^4x dy\sqrt{-g}\left[\frac{1}{2} g_{ab}\partial^a\phi\partial^b\phi - V(\phi)\right]. \tag{15}$$

And the energy-momentum tensor is given by

$$T_{ab} = -g_{ab}L + \partial_a\phi\partial_b\phi. \tag{16}$$

Because of the homogeneity, the nonzero space-like and time-like components must all be equal. Hence for 5-dimensional perfect fluid, we introduce two shapes of tensor, for SLED

$$T_b^a = diag(\rho, -P, -P, -P, -P^5). \tag{17}$$

And for TLED

$$T_b^a = diag(\rho, -P, -P, -P, \rho^5). \tag{18}$$

Because of homogeneity condition $\rho^5 = \rho$ and $P^5 = P$ are the density and pressure of the 5D perfect fluid respectively.

Because $T_{ab} = g_{a\beta} T_b^\beta$, one obtains from (1), (17) and (18)

$$T_{ab} = diag(e^{-2A(y)}\rho, e^{-2A(y)}P, e^{-2A(y)}P, e^{-2A(y)}P, P^5). \tag{19}$$

and

$$T_{ab} = diag(e^{-2A(y)}\rho, e^{-2A(y)}P, e^{-2A(y)}P, e^{-2A(y)}P, \rho^5). \tag{20}$$

Considering the bulk scalar field as a homogeneous perfect fluid, its energy-momentum tensor in (16) can be written in components as

$$T_{00} = e^{-2A}\left[\frac{1}{2}\phi'^2 + V(\phi)\right], \tag{21}$$

$$T_{ij} = -T_{00} = -e^{-2A}\left[\frac{1}{2}\phi'^2 + V(\phi)\right]\delta_{ij}, \tag{22}$$

$$T_{44} = \left[\frac{1}{2}\phi'^2 - V(\phi)\right], \tag{23}$$

for SLED [4] and

$$T_{00} = e^{-2A}\left[-\frac{1}{2}\phi'^2 + V(\phi)\right], \tag{24}$$

$$T_{ij} = -T_{00} = -e^{-2A}\left[-\frac{1}{2}\phi'^2 + V(\phi)\right]\delta_{ij}, \tag{25}$$

$$T_{44} = \left[\frac{1}{2}\phi'^2 + V(\phi)\right], \tag{26}$$

for TLED. Hear, the prime denotes the derivative with respect to only fifth coordinate y. Comparing the results (21), (22), (23) with (19) and (24), (25), (26) with (20), the consequent energy density and the pressure for the balk scalar field for SLED are

$$\rho = \left[\frac{1}{2}\phi'^2 + V(\phi)\right], \tag{27}$$

$$P = -\left[\frac{1}{2}\phi'^2 + V(\phi)\right], \tag{28}$$

$$P^5 = \left[\frac{1}{2}\phi'^2 - V(\phi)\right], \tag{29}$$

and for TLED are

$$\rho = \left[-\frac{1}{2}\phi'^2 + V(\phi)\right], \tag{30}$$

$$P = \left[\frac{1}{2}\phi'^2 - V(\phi)\right], \tag{31}$$

$$\rho^5 = \left[\frac{1}{2}\phi'^2 + V(\phi)\right]. \tag{32}$$

In the 5-dimensional warped geometry model, any particles and fields moving from the Planckbrane to the Tevbrane in the bulk would be growing, becoming lighter, and moving more slowly through time. Distance and time expand near the Tevbrane, and also mass and energy shrink near it [9–11]. The slow moving of the scalar field in the bulk can be similar to the slow rolling condition for inflaton field in standard 4D inflationary model. Therefore with this condition the potential energy of this field is larger than its gradient (Kinetic) energy ($\frac{1}{2}\phi'^2$), the negative pressure appears. These states, *i.e.* $P^5 = -\rho$ and $P = -\rho^5$ is very important in order to have inflation in 5D early universe ($\ddot{a} > 0$).

For 5D inflation, the fifth components of energy-momentum tensor, ρ^5 and P^5 is more important than other associated components in (19) and (20). Because on the Planckbrane, strings would be 10^{-33} cm in size, but on the Tevbrane, they'd be 10^{-17} cm. In fact, this makes the energy and mass scale for the Planckbrane based on about 10^{16} TeV . Therefor, the fifth components of pressure and density in compact size situate on the exited mode of Kaluza-Klein model and have an unstable state. This unstable and exited mode in 5D cosmic system soon can be undergoes a transition to a stable zero mode in 4D cosmic system, a phenomenon can be known as spontaneous symmetry breaking. Consequently, scalar field set in 4D universe and additional dimensions disappearance.

4 Conclusion

This paper provide an alternative mechanism for another inflationary scenario of early universe Versus 4D standard inflation. This mechanism produce of transition from unstable state of 5D world with Planck scale of energy to the 4D world with TeV scale of energy. It has been shown consideration 5D perfect fluid with SLED and TLED, can leads to negative pressure for energy-momentum tensor of the bulk scalar field. This condition is very important in order to have inflationary expansions for higher-dimensional world.

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